# MASTER OF SCIENCE / MASTER OF ART Syllabus & Structure M. Sc. (Statistics) (PGSTAT) / M. A. (Statistics) (MASTAT)

Semester	Course Code			Title of Papers			Credit	Max. Marks
1 <sup>st</sup> SEM	PGSTAT-101 / MASTAT -101			Measure and Probability Theory			4	100
	PGSTAT-102 / MASTAT -102			Statistical Inference		4	100	
	PGSTAT-103 / MASTAT -103			Survey Sampling			4	100
	PGSTAT-104(P) /			Practical based on PGSTAT- 101,102 and			6	100
	MASTAT -104(P)			103 / MASTAT-101,102 and 103				
Total Credit of 1 <sup>st</sup> Semester							18	
2 <sup>nd</sup> SEM	PGSTAT-105 / MASTAT -105			Linear Model and Design of Experiment			4	100
	PGSTAT-106 / MASTAT -106			Non Parametrics			4	100
	PGSTAT-	TAT -107	Stochast	Stochastic Process		4	100	
	PGSTAT-108 (P) /			Practical based on PGSTAT -105,106 and			6	100
	MASTAT -108 (P) 107 / MASTAT-105,106 and 107					-		
Total Credi	t of 2 <sup>nd</sup> Sem	nester					18	
3 <sup>rd</sup> SEM	PGSTAT-109 / MASTAT -109			Decision	Decision Theory and Bayesian Analysis		4	100
	PGSTAT-110 / MASTAT -110			Multivariate Analysis		4	100	
	PGSTAT-111 / MASTAT -111			Econometrics		4	100	
	PGSTAT-112 (P) /			<b>Practical</b> based on PGSTAT -109,110 and		6	100	
	MASTAT -112 (P) 111 / MASTAT-109,110 and 111							
Total Credit of 3 <sup>rd</sup> Semester							18	
4 <sup>th</sup> SEM	Compulsory Papers		PGSTAT-113 / MASTAT -113 Demography			4	100	
			PGSTAT-114 / MASTA		A <i>T -114</i>	Survival Analysis and Reliability Theory	4	100
			PGSTAT-115 / MASTAT -115 Actuarial Statistics		2	100		
	Select any one group							
	PGSTAT-116 / MASTAT -116 Operation Research					2	100	
	Group-I	PGSTAT-	STAT- 117 / MASTAT -11		Mathematical and Real Analysis		2	100
		PGSTAT-	118 (P) /			al based on PGSTAT-113	4	100
		MASTAT	-118 (P)		and 114	/ MASTAT-113 and 114		
	OR							
		PGSTAT-119 (DW) / MASTAT -119 (DW) Dissertation Work			4	100		
	Group-II	PGSTAT-120 (PV) / Practical based on PGSTAT-113, 114 and			PGSTAT-113, 114 and	4	100	
		MASTAT -120 (PV) Dissertation based viva voce / MASTAT-						
	113,114 and Dissertation based viva voce							
	Total Credit of 4 <sup>th</sup> Semester						18	
	Total Credit/Max. Mai							2000

# M.Sc. (Statistics) (PGSTAT) / M.A. (Statistics) (MASTAT)

#### Minimum Qualification of Admission:

The eligibility condition for admission in M.Sc. Program is that the candidate should pass the Any Bachelor Degree (10+2+2 OR 10+2+3 OR 10+2+4) with Mathematics/Statistics/Computer Science as one of the subject/paper and Mathematics in 10+2 level.

# <u>PGSTAT-101 / MASTAT-101</u> <u>Measure and Probability Theory</u>

### **Block – I:** Measure Theory

#### Unit - 1 Measure:

Field, -Field, Borel field. Measure, Meassure on R<sup>n</sup>, Properties of measure, Outer Measure, Extension of measures, Extension Theorem, Outer Extension. Simple functions, Integration, Non-negative integrable functions, Integrable measurable functions.

#### **Unit – 2: Convergence**:

Measure Space, Measurable Functions, Combinations of measurable function, point wise Convergence, Convergence in measure.

#### Unit – 3: Lebesgue Measure:

Lebesgue-Stielitjes measure, Lebesgue-stieltjes integral, Riemann-stieltjes integration, Lebesgue Dominated Convergence Theorem, Monotone convergence theorem, Fatou lemma, Fubini's theorem.

#### Unit – 4: Signed Measures:

Signed measures, Hahn and Jordan decomposition, Absolute Continuity, The Radon-Nikodym theorem, Derives of Signed Measures. Product Space, Cartesian products of two measurable spaces, Section, Product measures,

### **Block – II:** Probability Measure, Distribution Function and Inequalities

#### **Unit 1: Probability Measure:**

Probability space of a random experiment .probability measures, random variables as a measurable function. Field induced by a sequence of random variables,

#### **Unit 2: Distribution Functions:**

Decomposition of distribution functions in purely discrete, absolutely continuous and singular components.

#### **Unit 3: Probability Inequalities:**

CR-inequality, Chebyshev's inequality, Cauchy-Schwartz inequality, Holder inequality, Minkowski inequality, Jensen inequality, Lyapunov inequality, Kolmogorov inequality, Hajck-Renyki inequality.

## **Block – III:** Convergence, Characteristics Function and Limit Theorems

#### **Unit 1: Convergence:**

Sequences of distribution functions, weak and complete convergence of sequence of distribution function, Different types of convergence of sequence of random variables distribution function of random vectors,

#### Unit 2: Law of Large Numbers:

Weak law of large numbers (WLLN), Strong law of large numbers (SLLN), Khinchin's theorem, Borel zero-one law, Borel-Cantelli lemmas,

#### **Unit 3: Characteristic Function:**

Helly – Bray lemma and theorem, Weak compactness theorem, Kolmogorav theorems, Characteristic function, Inversion theorem, Continuity theorem, uniqueness theorem,.

#### **Unit 4: Central Limit Theorems:**

One dimensional central limit problem: lindeberg-levy, Lyapunov, Lindeberg-Feller theorems.

# PGSTAT-102 / MASTAT-102

# **Statistical Inference**

### **Block 1 - Estimation Theory**

#### **Unit 1: Point and Interval Estimation:**

Basic Concept of Point Estimation and Interval estimation, confidence level, unbiasedness, Criterion for Good Estimators, best linear unbiased estimator, relation between interval estimation and hypotheses testing.

### Unit 2: Sufficiency:

Sufficiency, factorization theorem, Fisher- Neyman – Halmos – Savage factorization criterion, minimal sufficiency and Ancillary statistics, invariance properties of sufficiency.

#### **Unit 3: Completeness:**

Completeness, Bounded completeness, Rao-Blackwell theorem, Lehman Schaffer theorem, Cramer-Rao inequality.

### **Unit 4: Exponential Family:**

Basu's theorem on independence of Statistics, Exponential families and Pitman families,

## **Block 2: Estimation, Hypothesis Testing and Confidence Estimation**

### **Unit 1: Methods of Estimation:**

Maximum likelihood estimation, method of moments, MVUE, necessary and sufficient conditions for MVUE, etc., Zehna theorem for invariance, Cramer theorem for weak consistency. Cramer-Huzurbazar theorem.

### **Unit 2: Criterion for Good Estimators:**

Criterion for Good Estimators, Bhattacharya bound, Chapman Robbins and Kiefer (CRK) bound, asymptotic normality, BAN and CAN estimators, asymptotic efficiency, equivariant consistency.

### **Unit 3: Confidence Estimation:**

Confidence interval and confidence coefficient, shortest length confidence interval, relation between confidence estimation and hypotheses testing.

### **Unit 4: Hypothesis Testing:**

Generalized Neyman Pearson lemma, MP and UMP tests for distributions with MLR, LR tests and their properties, UMPU tests, similar regions, Neyman structure, Invariant tests.

# PGSTAT-103 / MASTAT-103

Survey Sampling

## **Block 1 - Random Sampling Procedures - I**

### **Unit 1: Basics of Sampling Theory:**

Sampling Theory, sampling surveys vrs complete enumeration, types of sampling, sampling and non sampling errors.

#### **Unit 2: Simple random sampling:**

Sampling methods, SRSWOR and SRSWR, sampling for attributes.

### **Unit 3: Systematic sampling:**

Systematic sampling, Mean and variance of systematic sampling.

# **Block II - Random Sampling Procedures - II**

### Unit 1: Stratified Sampling and Use of Auxiliary Information:

Sampling Theory, stratified sampling, advantage of stratification, Post-stratification and deep stratification, Methods of allocation

### **Unit 2: Ratio and Regression Sampling:**

Ratio and Regression estimators, product method of estimation, double sampling in ratio estimation and double sampling in regression estimation, sub sampling.

### **Unit 3: Cluster and Multi-Stage Sampling:**

Cluster sampling with equal clusters, Cluster sampling with varying size of clusters, two stage sampling and multi-stage sampling.

### Unit 4: Response and Non Response Sampling:

Non sampling errors, Randomized Response Techniques (Warner's Model: related and unrelated questionnaire methods), ranked set sampling, controlled sampling, Non Response techniques, Non sampling errors with Non Response techniques.

# **Block III - Varying Probability Sampling**

## Unit 1: Methods of Selection and Ordered Estimators:

Varying probability sampling with and without replacement, cumulative total and Lahiri's methods of selection, Estimation of population mean.

## **Unit 2: Ordered Estimators:**

Concept of Ordered estimators, Desraj ordered estimates.

## Unit 3: Unordered Estimators:

Unordered estimator, Horvitz- Thompson estimator, Yates – Grundy modifications, Midzuno and Narain system of sampling.

# <u>PGSTAT—104 (P) / MASTAT-104 (P)</u>

Practical based on PGSTAT- 101,102 and 103 / MASTAT- 101,102 and 103

PGSTAT-105 / MASTAT-105 Linear Models and Design of Experiments

### **Block 1 - Linear Estimation and Analysis of Variance**

#### **Unit 1: Linear Model and BLUE:**

Linear Estimation- estimable functions, estimations and error space, Best linear unbiased estimate (BLUE), Markov theorem distribution of quadratic form, Estimable linear hypotheses generalized F and T tests.

#### Unit 2: Analysis of Variance- I:

Analysis of Variance : one-way and two-way classification with equal number of observation per cell and analysis with missing observations.

#### **Unit 3: Analysis of Variance- II:**

Analysis of Variance: one-way and two-way classification with unequal number of observation per cell, analysis with missing observations, Tukey's test general two-way classification, Analyses of covariance.

### **Block 2 - Design of Experiment**

#### **Unit 1: Basic Designs:**

Terminology and basic Principles of Design, CRD, RBD and LSD, analysis with missing observations.

#### **Unit 2: Factorial Experiments:**

 $2^3$ ,  $2^n$ ,  $3^2$  and  $3^3$  factorial experiments with its analysis.

### **Unit 3: Confounding:**

Orthogonality, Complete and Partial confounding, construction of confounded factorial experiments.

## **Block 2 – Advance Theory of Design of Experiment**

#### Unit 1: BIBD and PBIBD:

Balanced Incomplete Block Design (BIBD), Partially Balanced Incomplete Block Design (PBIBD), construction of BIBD and PBIBD, association schemes and construction, resolvable and affine resolvable design.

#### Unit 2: Split and Strip Plot Design:

Intra block and inter block analysis, Split Plot Design, Strip Plot Design.

#### **Unit 3: Other Advance Design:**

Dual and linked block design, Lattice Designs, Cross-over designs, optimal designsoptimal criteria, robust parameter design, response surface design – orthogonality, rotatability and blocking, weighing designs, mixture experiments.

# PGSTAT-106 / MASTAT-106

**Nonparametrics** 

### **Block 1- Order Statistics**

#### **Unit 1: Basic Distribution Theory:**

Order statistics, Distribution of maximum, minimum and r-th order statistic, Joint distribution of r-th and s-th order statistic.

#### **Unit 2: Asymptotic Distribution Theory:**

Moments of order statistics, asymptotic distributions of an order statistic, asymptotic relative efficiency, non parametric estimation of distribution function, Glivenko-Cantelli fundamental theorem.

#### **Unit 3: Distribution Free Intervals:**

Distribution of range function of order statistics, distribution free confidence intervals for quintiles, distribution free tolerance interval, distribution free bounds for moments, Fooleries limits.

#### **Unit 4: Rank order Statistics:**

Rank order statistics, Dwass' technique, Ballot theorem its generalization, extension and application to fluctuations of sums of random variables.

#### **Block 2- Sequential Analysis**

#### **Unit 1: Sequential Tests:**

SPRT and its properties, Wald's Fundamental identity, OC and ASN functions, Wald's equation, Wolfowitz generalization of FRC bound, Stein's two stage procedure.

#### **Unit 2: Sequential Estimation:**

Asymptotic theory of sequential estimation, sequential estimation of normal mean.

### **Block 3- Nonparametric Tests and Inference**

#### **Unit 1: One- sample Location Tests**

One and two sample location tests, Sign test. Wilcoxon test, Median test.

### Unit 2: Other non- parametric tests

Mann- Whitney U- Test, Application of U-statistic to rank tests. One sample and two sample Kolmagorov-Smirnov tests. Run tests.

### **Unit 3: Nonparametric Inference**

The Kruskal-Wallis one way ANOVA Test, Friedman's two-way analysis of variance by ranks, efficiency criteria and theoretical basis for calculating ARE, Pitman ARE.

# <u>PGSTAT—107 / MASTAT-107</u>

# **Stochastic Process**

# **Block 1: Types of Processes**

## Unit 1: Poisson Processes

Poisson (point) process, Brownian motion process, thermal noise, Markov short noise, two valued process, Model for system reliability, mean value function and covariance kernel of Poisson process, Increment process of a Poisson process, Stationary and evolutionary process.

### Unit 2: Branching Processes

Simple branching process, probability generating function, average size, varince and moments of number of individuals in the n-th generation, total progeny in branching process.

## Unit – 3: Wiener Process:

Wiener process, mean value function and covariance kernel of wiener process, Arc-sine law, Martingales, Stopping times, Optional sampling theorem.

## Unit – 4: Renewal Process:

Renewal process, distribution and asymptotic distribution of renewal process, elementary renewal theorem, delayed and equilibrium renewal process.

# **Block 2:** Markov Chains and Markov Process

## Unit 1: Markov Dependent Trials:

Two state Markov sequences, Markov chains, Markov classification of states and chain recurrent events, delayed recurrent events, application to the theory of success runs, more general patterns for recurrent events.

## Unit 2: Transition Probabilities:

Determination of n-step transition probabilities, Chapman-Kolmogorov equations, first return and first passage probabilities, fundamental theorem of probability of extinction, higher transition probabilities in Markov classification of states and chain.

### Unit 3: Classification of States:

Classification of states, communication states, periodicity, stationary probability distributions, limit theorems, Ergodic chains and Irreducible Ergodic chains.

### Unit 4: Continuous Time Markov Processes:

Markov processes in Continuous time. Interval arrival time, stopping time, optional stopping theorem, wald's equation, forward and backward equations for homogeneous case, random variable technique.

# **Block - 3: Random Walk and Queuing Process:**

### Unit 1: Random Walk and Gambler's Ruin Problem:

Random walk, Brownian motion as a random walk, one-dimensional, two-dimensional and three-dimensional random walks, duality in random walk and gambler's ruin problem.

### Unit 2: Queuing Process:

Birth and death processes, renewal process, Queuing models- Specification & Effectiveness, Measures, the  $E_k/M/1$ ,  $M/E_k/1$ ; M/M/1; M/M/k & M/G/1 queuing process.

## Unit 3: Distributions:

Compound distribution, Machine Interference Problem, Waiting Time Distribution for M/M/1 and M/M/k models,

### Unit 4: Martingales:

Martingales, Boob – Decomposition, Martingale convergence theorems.

# **Block - 4: Applied Stochastic Process:**

## Unit 1: Homogeneous Process:

Forward and backward equations for homogeneous case, random variable technique, homogeneous birth and death process, divergent birth process, the general birth and death process, multiplicative process, effect of immigration for homogeneous process.

## Unit 2: Non-Homogeneous Process:

Simple non homogeneous process, Polya process, effect of immigration for non homogeneous process, Diffusion, Backward Kolmogorov diffusion equation, Fokker-Planck equation.

### Unit 3: Non Markovian Process:

Some multi dimensional prey and predator, Non Markovian Process, Embedded Markov Process, Application to population growth, epidemic and counter models.

# <u>PGSTAT—108 (P) / MASTAT-108 (P)</u>

Practical based on PGSTAT- 105,106 and 107 / MASTAT- 105,106 and 107

# <u>PGSTAT—109 / MASTAT-109</u> <u>Decision Theory and Bayesian Analysis</u>

# **Block 1: Basic Elements and Bayes Rules**

### Unit 1: Basic Elements

Decision theoretic problem as a game, basic elements, optimal decision rules, unbiased ness, invariance, ordering.

### Unit 2: Bayes and Minimax Rules

Bayes and minimax principles, generalized. Bayes rules, extended Bayes rules, Limit of Bayes rule.

### Unit 3: Bayesian interval estimation:

Baysian interval estimation, credible intervals, HPD intervals, comparison with classic confidence intervals.

# **Block 2: Optimality of Decision Rules**

### Unit 1: Admissibility and Completeness:

Admissibility, completeness, minimal complete class, separating and supporting hyper plane theorems.

### Unit 2: Minimaxity and Multiple Decision Problems:

Minimax theorem, complete class theorem, equalizer rules and examples, multiple decision problems, continuous form of Bays theorem, its sequential nature and need in decision making.

### Unit 3: Bayesian Decision Theory:

Basic elements of Bayesian decision theory, theorem on optimal Bays decision function, relationship of bays and minimax decision rules, least favorable distributions.

### Unit 4: Bayesian inference:

Bayesian sufficiency, improper prior densities, Natural Conjugate Bayesian density (NCBD), posterior odd ratio, HPD regions, Bayesian inference for normal populations, empirical bayes procedures, bayesian testing of hypothesis.

# **Block 3: Bayesian Analysis**

### Unit 1: Prior and Posterior Distributions:

Subjective probability, its existence and interpretation, Prior Distribution, subjective determination of prior and posterior distribution, improper priors, non informative priors, invariant priors, conjugate prior families, construction of conjugate families using sufficient statistics of fixed dimension.

### Unit 2: Bayesian Inference Procedures:

Parametric empirical Bayes, Bayesian Inference, point estimation, credible sets, testing of hypothesis, Admissibility and minimaxity of Bays and Generalized bays procedures.

### Unit 3: Bayesian Robustness:

Ideas of Bayesian robustness, asymptotic expansion for posterior density, Baysian calculation, Monto carlo Integration and Markov chain Monto Carlo techniques.

# PGSTAT—110 / MASTAT-110 Multivariate Analysis

# **Block 1: Multivariate Normal Distribution and Estimation of Parameters**

### Unit 1: Multivariate Normal Distribution

Multivariate normal distribution, Moment generating function, Characteristic function, marginal and conditional distributions, multiple and partial correlation coefficient.

### Unit 2: MLE of Parameters and different coefficients

Maximum likelihood estimators of the mean vector and covariance matrix, sample Multiple and partial correlation coefficients, regression coefficient.

### Unit 3: Sampling Distributions

Distributions of sample mean vector, Null sampling distributions of Multiple and Partial Correlations, distribution of sample regression coefficient. Distribution of the matrix of sample regression coefficients and the matrix of residual sum of squares and cross products, Rao's U-statistic, its distribution and applications.

# **Block 2: Distributions Related to MND and their Applications**

### Unit 1: Wishart Distribution

Wishart distribution. Its characteristic function, additive property of Wishart distribution, Cochran theorem distribution of characteristic roots and vectors of wishart matrices.

# Unit 2: Hoteling's T<sup>2</sup> Statistic

Hoteling's  $T^2$  Statistic, Null distribution and non null distribution of Hoteling's  $T^2$  Statistic, Applications in tests for the mean vector of one and more multivariate normal population.

### Unit 3: Mahalnobis $D^2$

Equality of the component of a mean vector in a multivariate normal population, Mahalanobis  $D^2$  and its various applications.

### Unit 4: Discriminant Analysis

Discriminant analysis, classification and discriminatiuon procedures for discrimination betweentwo multivariate normal populations, sample discriminant function, tests associated with discriminant functions, probabilities of miss classification and their estimation, classification into more than two multivariate normal populations, Fuisher-Behren Problem

## **Block 3: Advance Multivariate Analysis**

### Unit 1: Advance Analysis

Inadmissibility of maximum likelihood estimator of mean vector of multivariate normal distribution when dimension is greater than three, James-Stein estimator of the mean vector and improved estimation of dispersion matrix of a MND.

### Unit 2: Principle Component Analysis

Principle components, Principle component analysis, their maximum likelihood estimators and sample variances, canonical correlation and variable, Interference on canonical correlations.

### Unit 3: Factor Analysis

Factor analysis, linear factor models, estimation of factor loadings, factor rotation, estimation of factor scores.

### Unit 4: Tests of Hypothesis

Tests of hypothesis of equality of covariance matrices, sphericity tests for covariance matrix, mean vector and covariance matrix are equal to given vector and matrix.

### Unit 4: Linear Regression Model

Multivariate linear regression model \_\_\_\_\_ estimation of parameters and their properties. Multivariate analysis of variance [MANOVA] of one-way classified data. Wilk's lambda criterion.

# PGSTAT—111 / MASTAT-111 Econometrics

### **Block 1: Linear Model and its generalizations**

#### Unit 1: Linear regression models:

Linear regression model. Assumptions, estimation of parameters by least squares and maximum likelihood methods. LOGIT, PROBIT, TOBIT and multinomial choice models, passion regression models.

#### Unit 2: Multicollinearity:

Multicolliearity, problem of multicollinearity, consequences and solutions, regression and LASSO estimators.

### Unit 3: Estimation of parameters and prediction

Testing of hypotheses and confidence estimation for regression coefficients,  $R^2$  and adjusted  $R^2$ , point and interval predictors.

#### Unit 4: Model with qualitative independent variables:

Models with dummy independent variables, discreet and limited dependent variables. Use of dummy variables, model with non-spherical disturbances, estimation of parametric by generalized equation.

#### Unit 5: Non-spherical disturbances

Seemingly unrelated regression equations (SURE) model and its estimation, Panel data models, estimation in random effect and fixed effect models.

## **Block 2: Simultaneous Equations Models and Forecasting**

### Unit 1: Structural and reduced form of the model and identification problem

Simultaneous equations model, concept of structural and reduced forms, problem of identification, rank and order conditions of indentifiability.

#### Unit 2: Estimators in simultaneous equation models

Limited and full information estimators, indirect least squares estimators, two stage least squares estimators, three stage least squares estimators and k class estimator.

## Unit 3: Estimation in simultaneous equation models

Limited information maximum likelihood estimation, full information maximum likelihood estimation, prediction and simultaneous confidence interval.

## Unit 4: Forecasting

Forecasting, exponential and adaptive smoothing methods, pereiodogram and correlogram analysis.

### Unit 5: Instrumental Variable Estimation

Review of GLM, analysis of GLM and generalized leased square estimation, Instrumental variables, estimation, consistency properties, asymptotic variance of instrumental variable estimators.

# **Block 3: Advance Econometrics**

### Unit 1: Autoregressive Process:

Moving average (MA), Auto regressive (AR), ARMA and ARMA models, Box-Jenkins models, estimation of ARIMA model parameters, auto covariance and auto correlation function.

### Unit 2: Vector Autoregressive Process:

Multivariate time series process and their properties, vector autoregressive (VAR), Vector moving average (VMA) and vector autoregressive moving average (VARMA) process.

### Unit 3: Granger Causality:

Granger causality, instantaneous Granger causality and feedback, characterization of casual relations in bivariate models, Granger causality tests, Haugh-Pierce test, Hsiao test.

### Unit 4: Cointegration:

Cointegration, Granger representation theorem, Bivariate cointegration and cointegration tests in static model.

# <u>PGSTAT—112 (P) / MASTAT-112 (P)</u>

Practical based on PGSTAT- 109,110 and 111 / MASTAT- 109,110 and 111

# PGSTAT-113 / MASTAT-113

# <u>Demography</u>

**Block -1.** Migration

### Unit-I

Introduction, Estimation of life time and inter-censal migration from place of birth statistics, estimation of internal migration from statistics on duration of residence, at a fixed poor date.

### Unit -2

Indirect measure of net internal migration based on growth rate method, methods to Estimate intercensal migration-using vital statistics, life time survival ratio method and census survival methods, estimation of international migration.

# **Block-2.** Stable Population Theory

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### Unit-1

Introduction, basic concepts of stable, quasi-stable, stationary and non-stable populations, vital rates and characteristics of stationary stable population and quasi-stable population.

### Unit-2

Definition of intrinsic rates of natural increase, intrinsic birth rate and intrinsic death rate, their relationship, derivation of Lotka's formulae of fundamental relationship instable population.

## Unit-3

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Computation of intrinsic rate of natural increase and construction of stable age distribution from the given fertility and mortality schedules, relationship between net reproduction rate(NRR), intrinsic rate of natural increase and mean length of generation, concept of mean interval between two generations.

# Block-3. Fertility & Fertility Models.

### Unit-1

Introduction, crude birth rate (CBR), gross fertility rate (GFR,) age specific fertility rate) ASFR), total fertility rate (TFR), gross reproduction rate (GRR)

## Unit-2

Period and cohort measures, use of birth order statistics, child women ratio, own-children method, children ever born(CEB) data and with data on current fertility, Brass P/F ration for adjusting fertility rates.

## Unit-3

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Simple model on time of first birth/conception and number of births/conception n specified time, birth interval models, study of fertility through birth interval analysis.

# **Block-4.:** Mortality

Unit-1

Introduction, crude death rate (CDR), specific death rates (SDR), standardized death rate (STDR).

Unit-2

Life table, abridge life table, model life table of UNO (old and new),coale and demny model, brass model through logit transformation.

# <u>PGSTAT-114 / MASTAT – 114</u> <u>Survival Analysis and Reliability Theory</u>

### **Block 1 – Survival Analysis**

#### **Unit 1: Basic Concepts:**

Concepts of time, Order and random Censoring, likelihood in these cases. Types of Censoring and truncation, Life tables, failure rate, mean residual life and their elementary properties. Ageing classes - and their properties, Bathtub Failure rate. Estimation of survival function - Acturial Estimator, Kaplan - Meier Estimator, log rank tests,

#### **Unit 2: Parametric Survival Models:**

Assumptions and Characteristics, Life distributions-Exponential Gamma, Weibull, Lognormal, Pareto, Rayleigh, piece-wise exponential etc, Linear Failure rate. Parametric inference (Point estimation, Confidence Intervals, Scores, LR, MLE tests (Rao-Willks-Wald)) for these distributions. Estimation under the assumption of IFR/DFR.

#### **Unit 3: Non-Parametric Survival Models:**

Assumptions and Characteristics, of exponentiality against non-parametric classes- Total time on test, Deshpande test. Two sample problem-Gehan test, Log rank test. Mantel-Haenszel test, Tarone – Ware tests.

#### **Unit 4: Proportional Hazard Models:**

Assumptions and Characteristics, Semi-parametric regression for failure rate - Cox's proportional hazards model with one and several covariates. Rank test for the regression coefficients. Competing risks model, parametric and non-parametric inference for this model. Multiple decrement life table.

### **Unit 5: Recurrent Event Survival Analysis:**

Introduction, Outline and Objective, competing risks survival Analysis, competing risk events and Frailty models

# **Block 2 – Reliability Analysis**

#### **Unit 1: Basic Concepts:**

Reliability concepts and measures; components and systems; coherent systems; reliability of coherent systems; cuts and paths; modular decomposition; bounds on system reliability; structural and reliability importance of components.

### Unit 2: Ageing:

Concept of Ageing, Ageing classes - and their properties, Notions of ageing; IFR, IFRA, NBU, DMRL, and NBUE Classes and their duals;

### **Unit 2: Reliability Estimation:**

Reliability estimation based on failure times in variously censored life tests and in tests with replacement of failed items; stress-strength reliability and its estimation.

### Unit 3: Repairable Systems:

Maintenance and replacement policies; availability of repairable systems; modeling of a repairable system by a non-homogeneous Poisson process, preventive maintenance policy, preliminary concepts of coherent systems.

### **Unit 4: Growth Models and Accelerated Life Testing:**

Reliability growth models; probability plotting techniques; Hollander-Proschan and Deshpande tests for exponentiality; tests for HPP vs. NHPP with repairable systems. Basic ideas of accelerated life testing.

# <u>PGSTAT-115 / MASTAT – 115</u>

# Actuarial Statistics

# **Block I- Probability Models and Life Tables**

### Unit 1:

Utility theory, insurance and utility theory, models for individual claims and their sums, survival function, curtate future lifetime, force of mortality.

## Unit 2:

Life table and its relation with survival function, examples, assumptions for fractional ages, some analytical laws of mortality, select and ultimate tables.

#### Unit 3:

Multiple life functions, joint life and last survivor status, insurance and annuity benefits through multiple life functions evaluation for special mortality laws.

### Unit 4:

Multiple decrement models, deterministic and random survivorship groups, associated single decrement tables, central rates of multiple decrement, net single premiums and their numerical evaluations.

### Unit 5:

Distribution of aggregate claims, compound Poisson distribution and its applications.

# **Section II- Insurance and Annuities**

## Unit 1:

Principles of compound interest. Nominal and effective rates of interest and discount, force of interest and discount, compound interest, accumulation factor, continuous compounding.

### **Unit 2:**

Life insurance: Insurance payable at the moment of death and at the end of the year of death-level benefit insurance, endowment insurance, diferred insurance and varying benefit insurance, recursions, commutation functions.

### Unit 3:

Life annuities: Single payment, continuous life annuities, discrete life annuities, life annuities with monthly payments, commutation functions, varying annuities, recursions, completeannuities-immediate and apportionable annuities-due.

## Unit 4:

Net premiums: Continuous and discrete premiums, true monthly payment premiums, apporionable premiums, commutation functions, accumulation type benefits. Payment premiums, apportionable premiums, commutation functions, accumulation type benefits.

## Unit 5:

Net premium reserves: Continuous and discrete net premium reserve, reserves on a semicontinuous basis, reserves based on true monthly premiums, reserves on an apportionable or discounted continuous basis, reserves at fractional durations, allocations of loss to policy years, recursive formulas and differential equations for reserves, commutation functions.

## Unit 6:

Some practical considerations: Premiums that include expenses-general expenses types of expenses, per policy expenses. Claim amount distributions, approximating the individual model, stop-loss insurance.

*PGSTAT-116 / MASTAT – 116* 

**Operation Research** 

# Block – 1: Linear & Non-Linear Programming

- Unit 1 Review of LP Problems, Methods of Solution, Duality Theorem, Transportations & Assignment Problems with Proof of Relevant Results
- Unit 2 Methods Using Artificial Variables, Two Phase and Penality, Degeneracy & Cycling, Sensitivity Analysis.
- Unit 3 Non-Linear Programming, Kutin Tucker Theorem, Wolfe's and Beale's Algorithm for Solving Quadratic Programming, Bellman's Principle of Optimality

# Block – 2: Theory of Games & Sequencing & Network Analysis

- Unit 1Games in Normal and Extended forms, Fundamental Theorem of Matrix Games,<br/>Solution of 2x2. 2xm and Mxn Zero-sum games by Dominance Principles.
- Unit 2 Sequencing and Scheduling Models, 2 Machin, n-Job Problem (no passing), 3 machine, n-job problems, different routing- 2 jobs & m stations, travelling sales-man problem.
- Unit 3 Introduction to networks, determination of flows and of critical paths, CPM & PERT

# **Block - 3: Queuing Theory**

- Unit 1Queuing models- Specification & Effectiveness Measures, the  $E_k/M/1$ ,  $M/E_k/1$ ;<br/>M/M/1; M/M/c & M/G/1 Queses, and their Steady State Solutions
- Unit 2 Machine Interference Problem, Waiting Time Distribution for M/M/1 and M/M/C models

# **Block – 4: Replacement Problems**

- Unit 1 Replacement Problems, Replacement of items that Depreciate, Discounted Cash Flow in Investment Problems.
- Unit 2 Replacement of items Failing According to a Probability Law; block and age replacement policies, Staffing Problem, Dynamic Programming Approach for Maintenance Problems.

# PGSTAT-117 / MASTAT-117

# Mathematical and Real Analysis

# **Block 1- Riemann Stieltjes Integrals, Fourier Series and Functions of Bounded** Variation

### **Unit 1: Riemann Stieltjes Integrals:**

Absolutely continuous functions. Riemann Stieltjes integrals. Basic theorems. Definitions, Linear properties, integration by parts, change of variable in . Riemann Stieltjes integrals, upper and lower integrals, necessary and sufficient conditions for existence of . Riemann Stieltjes integrals, integral as a function of parameters, differentiation under the integral sign.

#### **Unit 2: Fourier Series:**

Fourier Series, orthogonal system of functions, Fourier series of a function relative to an orthogonal system, properties of Fourier Coefficients, Reusz- Fischar theorem, convergence and representation problems for Fourier Metric Series, Sufficient conditions for convergence of Fourier Series at a particular point.

#### **Unit 3: Bounded Variation:**

Functions of bounded variation, total variation, function of bounded variation expressed as the difference of increasing functions, continuous functions of bounded variation, Absolutely continuous functions.

### **Block 2- Metric Spaces & Continuity**

#### **Unit 1: Metric Spaces:**

Metric Spaces, open and closed sets, limit and cluster points, Cauchy Sequences and completeness, Convergence of sequences, Completeness of R". Baire's theorem. Cantor's ernary set as example of a perfect set which is now here dense.

#### **Unit 2: Continuity:**

Continuity and uniform continuity of a function from a Metric space to a Metric space. Open and closed maps, Compact spaces and compact sets with their properties. Continuity and compactness under continuous maps.

#### **Unit 3: Analytic Functions and Transformation:**

Analytic function, Cauchy-Riemann equations, Cauchy equation formula, its applications, Fourier and Laplace transforms.

#### **Block –3:** Real Analysis

### **Unit – 1: Basic Concepts:**

Recap of elements of set theory; Introduction to real numbers, Introduction to n-dimensional Euclidian space; open and closed intervals (rectangles), compact sets, Bolzano - Weirstrass theorem, Heine – Borel theorem.

### Unit -2: Sequences and Series:

Sequences and series; their convergence. Taylor's Series, Real valued functions, continuous functions; uniform continuity, sequences of functions, uniform convergence; Power series and radius of convergence, Singularities, Laurent Series.

### Unit – 3: Integration:

Differentiation, maxima - minima of functions; functions of several variables, constrained maxima - minima of functions, Multiple integrals and their evaluation by repeated integration. change of variables in multiple integration. Uniform convergence in improper integrals, differentiation under the sign of integral - Leibnitz rule, Residue and contour integration.

# <u>PGSTAT—118 (P) / MASTAT-118 (P)</u>

Practical based on PGSTAT- 113 and 114 / MASTAT- 113 and 114

# PGSTAT-119 (DW) / MASTAT-119 (DW)

**Dissertation Work** 

# PGSTAT-120 (PV) / MASTAT-120 (PV)

Practical based on PGSTAT- 113, 114 and Dissertation based viva voce / MASTAT- 113, 114 and Dissertation based viva voce