Course Structure FOR
Choice Based Credit System of
B.Sc. (Mathematics) Program with effect from 2020-21
School of Science, UPRTOU, Prayagraj

Semester	Course Code	Title of Paper	Credits	Max. Marks
I	UGMM-101	Differential Calculus	2	100
	UGMM-102	Analytical Geometry	2	100
II	UGMM-103	Integral Calculus	2	100
	UGMM-104	Differential Equation	2	100
III	UGMM -105	Mechanics-I (Statics and Dynamics)	2	100
	UGMM -106	Mechanics-II (Dynamics and Hydrodynamics)	2	100
IV	UGMM -107	Linear Algebra	2	100
	UGMM -108	Calculus of function of several variable and Vector Calculus	2	100
V	**Discipline Centric Elective Course**			
	DCEMM -109	Abstract Algebra	2	100
	DCEMM -110	Number Theory	2	100
	DCEMM -111(P)	Viva Voce	2	100
	Skill Enhancement Course			
	SBSMM-01	Elementary Analysis	4	100
VI	**Discipline Centric Elective Course**			
	DCEMM -112	Advance Analysis	2	100
	DCEMM -113	Function of Complex Variable	2	100
	DCEMM-114(P)	Viva Voce	2	100
		Total Credit	**32**	**1500**

B.Sc.
SYLLABUS of MATHEMATICS

UGMM-101: Differential Calculus

Block-I

Set, Relation, Function and its Property

Unit 1 Set and Relation Set Theory, Types of sets, Operations on Sets, Laws Relating Operations, De Morgan's Laws, Venn diagram, Cartesian product of two sets, Relation, Definition and Examples Domain and Range of a Relation, Types of Relations, Composition of Relation, Equivalence relation in a set, Partition of a Set, Quotient set of a set, Oder Relation and Examples.

Unit-2 Functions or mapping, Direct and inverse images of subsets under maps, Real valued Functions of one variable, Inverse functions, Graphs of functions, Operations on functions, Composite of functions, Even and odd functions , Monotone functions, Periodic functions, Axiomatic introduction of $\mathbb{R}$ as a complete ordered field, Basic properties of R, Absolute value, Intervals on the real line

Unit-3 Limits Definition of limit of a function at a point of its domain, Algebra of Limits, Infinite Limits (Limits as $x \to \pm\infty$), One Sided Limits.

Unit-4 Continuity (Definitions and Examples), Algebra of continuous functions, Properties of continuous functions, Local Boundedness supremum and infimum of a function, Boundedness and intermediate value theorem, properties of continuous functions over closed intervals, Type of discontinuity, Image of a closed interval under continuous maps.

Block 2

Differential Calculus

Unit-5 Differentiability and Derivatives Differentiability of a function at a point, Definition of derivative of a function and its geometrical interpretation, Derivatives of some simple functions, Algebra of derivatives, Chain rule. Sign of derivatives and monotonicity of functions, Continuity versus Differentiability. Derivative of exponential function, Logarithmic functions.

Unit-6 Derivative of Hyperbolic Functions and Some Special Functions Definition of Hyperbolic Functions, Derivative of Inverse Hyperbolic Functions Methods of Differentiation (Derivative of x^r), Logarithmic Differentiation, Derivatives of functions defined in terms of a parameter, Derivatives of Implicit Functions , Derivatives of Trigonometric Functions, Derivative of the Sine Function, Derivative of the Cosine Function, The Derivatives of the other trigonometric functions, Derivative of the Tangent Function, Derivatives of Inverse Functions, Derivatives of Inverse Trigonometric Functions, Use of Transformations

Unit-7 Successive Differentiation Second and Third order Derivatives, n^{th} Order Derivatives. Leibnitz's Theorem. Maclaurin's Series, Taylor's Series

Unit-8 Mean value theorems Rolle's theorem, Lagrange's Mean value Theorem and Cauchy Mean value Theorem.

UGMM-102: Analytical Geometry

Unit 1:- Homogeneous equation of second degree and conditions on it to represent different types of conics.

Polar coordinates: Polar equation of a line, parabola, ellipse and hyperbola when focus is taken as pole. Polar equations of the chord joining two points, Tangent, normal, polar (chord of contact), pair of tangent lines, asymptotes, Tracing of a conic.

Unit 2:- Straight line and plane, direction cosines and direction numbers, distance of a point from a line, various form of the equation of a plane, plane passing through three given points, angle between two lines and two planes, distance of a point from a plane, equation of line of intersection of two planes, intersection of line and plane. coplanar lines shortest distance between two skew lines.

Unit 3:- Equation of a sphere, Intersection of sphere and planes, Intersection of two sphere. Sphere passing through a circle, Intersection of a straight line and a sphere. Tangent planes, Polar planes, Plane of contact. Power of a point. Radical planes, Radical lines, Co-axel system of a sphere. Orthogonal system of sphere.

Unit 4: Cones: Equation of a cone with a given base, Intersection of a cone and a plane passing through the vertex of cone, tangent plane, reciprocal cone, Enveloping cone, right circular cone.

Unit 5: Cylinders: Equation of a cylinder with given base, Cylinder with Axis parallel to co-ordinate axes. Enveloping cylinders, Right circular cylinders.

Ruled surfaces, generating lines of a hyperboloid of one sheet and their simple properties.

Unit 6:- Central Conicoids: Standard equation of a Central conicoid, ellipsoid, hyperboloid of one sheet and two sheets, tangent planes, tangent lines, polar planes and polar lines. Enveloping cones and cylinders section with a given centres. Diametral plane, conjugate diameters, normal, normal drawn from a given point.

UGMM-103: Integral Calculus

Block 1: Integration

Unit 1- : Methods of integration:- Standard Integrals, Algebra of Integrals , Integration by Substitution, Integrals using Trigonometric formula, Trigonometric and Hyperbolic Substitution, Two properties of Definite integrals, Integration by Parts, Evaluation of $\int (a^2 - x^2)\, dx$, $\int (a^2 + x^2)\, dx$, $\int (x^2 - a^2)\, dx$, $\int e^{ax}$, $[f(x) + f'(x)]\, dx$.

Unit-2 Reducation Formulas Reduction formula , Integrals Involving trigonometric functions , Integrals involving products of trigonometric functions, Integrals Involving Hyperbolic Functions.

Unit-3 Integration of Rational and Irrational Functions Integration of Rational and Irrational Functions Integration of Rational Function , Some simple Rational Function , Partial Fraction Decomposition, Method of Substitution, Integration of Rational Trigonometric Functions, Integration of Irrational Functions

Unit-4 Tangent Normal of the curves| Equations of tangents and normal, Angles of intersection of two curves, Tangents at the origin.

Block-II Application of Integral Calculus

Unit-5 Tracing of curves:- Classifying singular points, Asymptotes (Parallel to the axes and oblique asymptotes. Tracing of curves.

Unit-6 Area Under a Curve Area of the curve in Cartesian form, Polar form, Area Bounded by a closed curve**,** Length of a Plane Curve in Cartesian Form, Parametric Form, Polar form,

Unit-7 Volume of a solid of Revolution:- Volume of a solid of Revolution in Parametric Form, Polar form, Area of Surface of Revolution in Cartesian Form, Parametric Form and Polar form.

UGMM-104: Differential Equation

Block-I
Differential Equations of First Order and First Degree

Unit 1: Differential equation, Types of differential equations, Order and degree of differential equations, Formation of differential equation, Solution of differential equation, Geometrical meaning of a differential equation, Initial value problems and statement of Existence and Uniqueness Theorems.

Unit 2:- Methods of solution of a differential equation of first order and first degree: Method of separation of variables. Solution of homogeneous equations. Equation reducible to homogeneous form.

Unit 3:- Linear differential equation and Bernoulli's linear differential equation.

Unit 4: Exact differential equations. Integrating factors to solve non exact differential equations in different cases.

Unit 5: Differential equation of the first order but not of the first degree, Equation solvable for x, y and p. Clairaut's equation and singular solutions.

Block-II
Applications of differential equation

Unit 6:- Applications of differential equation, geometrical application and physical applications; Newton's law of cooling, Kirchoff's law of electric circuits, motion under Gravity, rectilinear motion, simple harmonic motion, rate of growth or decay, heat flow.

Block-III
The n^{th} order linear differential equation with constant coefficients

Unit 7:- The n^{th} order linear differential equation with constant coefficients, general solution and particular integrals. Method of finding particular integrals Methods of undetermined coefficient, variation of parameters.

Unit 8:- Method of finding particular integrals by inverse operator methods.

Unit 9:- Equation reducible to Linear with constant coefficients, Euler-Cauchy linear equations. Simultaneous linear differential equation with constant coefficient.

Unit 10:- Linear differential equations of second order. Transformation of the equation by changing the dependent variable, independent variables and Normal forms.

UGMM-105: Mechanics-I (Statics and Dynamics)

Statics

Unit 1:- Common Catenaries

Unit 2:- Virtual work and

Unit 3:- Stable and unstable equilibrium.

Dynamics of a particle

Unit 4:- Motion in a plane : Velocities and accelerations in Cartesian coordinates, along radial and transverse directions, and along tangential and normal directions. Determination of path under a given force.

Unit 5:- Rectilinear Motion: Simple harmonic motion. Elastic strings. Motion under inverse square law and other miscellaneous laws, Motion in resisting medium. Motion of particles of varying mass, Rocket motion,

Unit 6:- Constrained motion (Vertical circle and vertical cycloid).

Unit 7:- Motion under Central forces: Central orbit, Conservation of angular momentum, areal velocity, Kepler's laws of motion, differential equations to the path of a particle,

UGMM-106: Mechanics-II (Dynamics and Hydrodynamics)

Unit 1: Moment of Inertia: Moment and product of inertia of some standard bodies, principle axis, Momental ellipsoid of a body.

Unit 2: D' Alembert Principle: The general equation of motion, motion of the centre of inertia and motion relative to the centre of inertia.

Unit 3: Motion about a fixed axis: Moment of the effective forces about the axis of rotation, moment of momentum about the axis of rotation, kinetic energy of the body rotating about a fixed axis, equation of motion about axis of rotation.

Unit 4: Equation of continuity in different coordinate system and boundary surfaces, velocity potential, stream lines.

Unit 5: Euler's equation of motion, steady motion, Bernaullies equation, Helmholtz equation, Impulsive motion.

Unit 6: Motion in two dimensions, stream function, irrotational motion, complex potential, sources and sinks.

Unit 7: Doublets, image system of a simple source with respect a plane, a circle, a sphere. Image system of a doublet with respect to a plane, a circle and a sphere, circle theorem.

UGMM-107: Linear Algebra

Unit 1:- Binary operations, Field, Examples: $\mathbb{Q}, \mathbb{R}, \mathbb{C}$**,** $\mathbb{Z}_p$ **(**p, a prime) of fields. Definition of a vector space, Some basic properties of a vector space, Vector subspace , Subspace spanned by a subsets with examples. Linearly dependence and independence of a subset.Finite dimensional vector spaces,

Unit 2:- Basis of a vector space, Dimension of a vector space, finite dimensional vector space, linear sum and direct sum of subspaces, Quotient spaces.

Unit 3:- Linear transformations:- Definition examples and some properties, Linear transformations and Isomorphism of vector spaces, Null space and range space. Rank and Nullity of a linear transformations, Fundamental theorem of vector space homomorphism, Rank-Nullity theorem, Non singular, Invertible transformations.

Unit 4:- Hom (V, W) as a vector space, dual space $\mathbf{V}^*$ of a vector space **V**, Dual basis of a vector space with examples, Transpose of a linear transformations. Annihilator of a subset of a vector space, Rank of transpose of a linear transformations.

Unit 5:- Matrix representation of a linear transformations, equality, Algebra of matrices, Multiplication of matrices. Vector space of all m × n matrices over a field. Ring of all n- square matrices. Invertible matrices (or Non-singular matrix), Transpose of matrix, Equivalent matrices, Similar Matrices, and Orthogonal matrices.

Unit 6:- Rank of a matrix, Row rank and column rank of a matrix, Elementary matrices, elementary row and column operations of a matrix , elementary matrices, Normal form of a matrix, Echelon form of matrix, Inverse of a non singular matrix. Determinantal rank of a matrix. Non homogeneous and homogeneous linear equations.

Unit 7:- Determinant of a square matrix, Cofactor of an element of a determinant, Properties of a determinant, minor of an element of a determinant, Evaluation of a determinant. Laplace expansion of a determinant. Product of two determinants(all statements without proof)

Unit 8:- Adjoint of a n- square matrix. Inverse of a matrix of a non-singular matrix by using adjoint of the matrix. Characteristic roots or Eigen values of a linear transformation and Eigen vector and Eigen space.

Unit 9:- Characteristic polynomial of a matrix, Diagonalization of a matrix, Caley - Hamilton theorem. Inverse of a matrix of a non singular matrix by Caley – Hamilton theorem. Characteristic polynomial of a linear transformation, Minimal polynomial, Hermitian Matrix, characteristic roots of a complex |Hermitian matrix.

Unit 10:- Inner product space: Definition and examples of inner product, length of a vector, Cauchy Schwarz inequality, distance between two vectors, angle between two vectors, Orthogonal and ortho normal sets.

Unit 11:- Bilinear, quadratic and Hermitian forms, bilinear form on a vector space **V**. Quadratic forms, matrix of quadratic form, normal form or canonical form.

UGMM-108: Calculus of function of several variable and Vector Calculus

Block-I Partial Differentiation

Unit 1:- **Partial Differentiation:-**Partial Derivatives of first order, Partial Derivatives of Higher order, Total derivative, Homogeneous functions, Euler's Theorem on Homogeneous functions, Deductions from Euler's Theorem.

Unit 2:- **Composite Functions**:- Differentiation of Composite Functions, Jacobians, Properties of Jacobians, Theorems on Jacobians, Jacobian of Implicit Functions, Functional Relationship,

Unit 3:- **Maxima and minima:-** Maxima and minima of functions of Two variables, Conditions for maxima and minima, Necessary condition for existence of maxima and minima, Local and global maxima and minima of a function, Lagrange's method of undetermined multipliers,

Block-II
Vector Calculus

Unit 4:- **Operations on Vectors:-** Scalar triple product and its geometrical interpretation. Properties of scalar triple product. Reciprocal system of vectors. Properties of the reciprocal system of vectors. Scalar product of four vectors. Vector product of four vectors

Unit 5:- **Differentiation of a Vector Functions and Gradient :-** Scalar and vector point function. Differentiation of a vector functions with respect to scalar. Gradient of a scalar field, geometrical interpretation of gradient, directional derivative, properties of gradient, gradient in polar coordinates.

Unit 6:- **Divergence and Curl:-** Divergence of a vector point function, Physical interpretation of divergence, Solenoidal vector. Curl of vector point function, Physical interpretation of curl, Irrotational vector, Vector identities, Velocity potential, Laplace operator.

Unit 7:- **Integration of a Vector Function:-** Line Integral, Surface Integral, Volume Integral.

Unit 8:- **Green's Theorem in a plane**, Gauss Divergence Theorem and Stokes's Theorem (Without proof) and their applications.

DCEMM -109: Abstract Algebra

Unit 1:- Elementary group theory:- Definition of a group, abelian groups examples including $\mathbf{Z_m}$, $\mathbf{Z_p}$, $\mathbf{U_m}$ the group of n^{th} roots of unity, Hamiltonian group, Klein's four group, Permutation group, Integral power of an element of a group, order of an element of a group.

Unit 2:- Subgroups of a group and examples. Homomorphism, isomorphism, Subgroup generated by a subset of a group. Cyclic groups.

Unit 3:- Coset decomposition, left coset and right coset of a subgroup of a group. Lagrange theorem. Index of a subgroup. Eulers theorem. Fermat's theorem.

Unit 4:- Normal subgroups, Centre of a group. Conjugate elements, Normaliser of an element of a group. Kernel of a homomorphism. Direct and inverse image of a subgroup and a normal subgroup under a homomorphism. Quotient groups.

Unit 5:- Fundamental theorem of homomorphism of groups. Symmetric group $\mathbf{S_n}$ .Cayley's theorem. Cycles transposition, Decomposition of a permutation, alternating groups $\mathbf{A_n}$. Automorphism of a groups. Inner automorphism.

Unit 6:-Ring Definition and examples. elementary property of a ring. zero divisor, Ring with or without zero divisor. Integral domain. Division ring. Field. Homomorphism and isomorphism of Rings, subrings, subfield with examples. Kernel of a homomorphism.

Unit 7 :-Direct image and inverse image of a subring and a subfield under a homomorphism. Characteristic of a non-zero integral domain. Imbedding of a ring into another ring the field of fractions of an integral domain..

Unit 8:- Ideals, left ideal and right ideal, principal ideal with example. Prime ideal, maximal ideal, Quotient rings. Fundamental theorem of homomorphism and rings and field.

DCEMM-110: Number Theory

Unit 1: Division algorithm, Euclid's algorithm for the greatest common divisor, Prime numbers, fundamental theorem of arithmetic

Unit 2: Linear congruences and algorithm to find the solution of Linear congruences, Chinese Remainder Theorem, An extension of Chinese Remainder Theorem (with non-coprime moduli).

Unit 3: Definition of Euler function, examples and properties, Multiplicative property of Euler's function.

Unit 4: The group of units modulo an integer, primitive roots, Existence of primitive roots, Quadratic congruences, Quadratic residues, Legendre symbol, Euler's criterion.

Unit 5: Gauss lemma, Gauss reciprocity theorem, Quadratic residues for prime-power moduli and arbitrary moduli

Unit 6: Arithmetic Functions**,** multiplicative functions and their properties, Mobius function and its properties, Mobius inversion formula and its applications.

SBSMM -01: Elementary Analysis

Unit 1:- Language of Mathematics, Mathematical statements, logical connectives, Tautology, quantifiers.

Unit 2:- Relations (definition and examples), types of relation, composite of relations, equivalence relation, equivalence class, partition of a set and order relation.

Unit 3:-Mapping (definition and examples), types of map, inverse map, composition of maps, direct and Inverse images of a set

Unit 4:- Real number system: Axiomatic definition of real number system as a complete ordered field. Archimedean principle. relational and irrational density theorem.

Unit 5:-Division in **Z:** Division algorithm, greatest common divisor, and least common multiple. Euclidean algorithm. Prime integers. Fundamental theorem of arithmetic's.

Unit 6:- Sequences, bounded and unbounded sequences, Subsequence, convergent, divergent and oscillatory sequences. Limit of a sequence. Algebra of convergent sequences, Cauchy's sequences, and Cauchy's criterion for convergence of a sequence.

Unit 7:- Partial sums of a series. Convergence and divergence of series. Series of nonnegative terms. Necessary and sufficient condition for convergence. P-series theorem. Comparison tests. D'Alembert's ratio test, Raabe's ratio test, Logarithmic test, Cauchy's condenses test and Root test, Alternating series. Leibnitz's theorem. Absolute convergence and conditional convergence of a series.

Unit 8:-Double and Triple integrals, Change of order of integration, surface and volume integration and their applications in areas and volume.

DCEMM -112: Advance Analysis

Unit 1:- Metric space (definitions and examples), open and closed balls, interior points, exterior point, and boundary points, limit points. open and closed sets, limit of a sequence in a metric space. Cauchy sequence.

Unit 2:- Limit and continuity of a function between metric spaces. Characterization of continuity in terms of open sets, Closed set and closer of a set.

Unit 3:- Compactness of metric space, Bolzano Weierstrass property, Total boundedness, sequentially compact metric spaces and countable compact metric space

Unit 4:- Uniform continuity, Lebesgue number, Complete metric space.

Unit 5:- Point wise and uniform convergence of a sequence and series of a functions, necessary and sufficient condition for a uniform convergence, Weierstrass test, Abels test

and Dirichlet's test for uniform convergence, Term by term integration and term by term differentiation .

Unit 6:- Convergence of improper integrals; Integral over infinite interval with bounded integrands and intervals over finite intervals with unbounded integranls. Necessary and sufficient conditions for such integrals.

Unit 7:- Comparison test, μ-test, absolute convergence, convergence of integrals of product of two functions, Abel's test, Dirichlet's test.

Unit 8:- Step function and their integrals, upper and lower integrals of a bounded function of one variables (through step functions). Integrable functions, Riemannian condition of integrability . Properties of Integrales of a step functions.

Unit 9:- Mean value theorem for integrals , Fundamental theorem of integral calculus. Primitive of a funcution. Change of variables, second mean value theorem (statements only)

DCEMM -113: Function of Complex Variable

Unit -1 Function of Complex Variable:- The concept of a function of a complex variable, Continuous functions, uniform continuity, bounded functions, differentiable and analytic functions, differenciabilty, analytic or regular functions Cauchy Riemann equations, necessary and sufficient condition for a function to be analytic, construction of analytic function, Milne Thomson method

Unit -2 Power Series: - The circle of convergence of a power series, power series and analytic functions, the exponential functions, the trigonometric functions, the logarithmic functions.

Unit -3 Complex Integration :- Jordan Arcs, Rectifiable arcs, countours, Complex integration, integration along a regular Arc, Cauchy theorem, the elementary form of Cauchy theorem, the general form of Cauchy theorem, extension of Cauchu's theorem on contours, defining multiply connected regions, Cauchy integral formula, derivative of an analytic function, Morera's Theorem.

Unit -4 Expansion in series and singularities :- Taylor series, Cauchy's inequalities, Liouville's theorem, Lourent's series, isolated singularites of an analytic function, the zeros of analytic function, the behaviour of analytic function at isolated singularities, limiting point of zeros or poles, the behaviour of an analytic function near an isolated essential singularity.

Unit -5 The calculus of Residues: - The Residue at a singularity, Residue at infinity, calculation of reduced in some special cases, Cauchy's theorem of Residues, poles and zeros of a Meromorphicfic function. Rouch's theorem, applications of Rouch's theorem,

Unit -6 Evaluation of Definite Integrals by Contour Integration:- Jensen's Theorem, Poisson's integral formula, The evaluation of integrals of the type $\int_0^{2\pi} f(\cos\theta, \sin\theta)d\theta$, The evaluation of integrals of the type $\int_{-\infty}^{\infty} f(x)dx$.

Unit -7 Conformal Representation:- Mappings by analytic functions, Differentiable arc through a point, conformal mapping, existence of the inverse function, Conformal character and analyticity, mapping by simple functions, super ficial magnification, The linear transformation, The Mobius (bilinear) transformation, The transformation $\omega = \frac{1}{z}$, Geometrical inversion, fixed points of a bilinear transformation.